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HEAT TRANSFER BETWEEN SOLID PARTICLES AND A GAS IN A NON-UNIFORMLY AGGREGATED FLUIDIZED BED

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Аннотация—На основе модельного представления о микропрорывах газа показана закономерность сочетания высоких истинных и много более низких эффективных коэффициентов теплообмена газа с материалом в неоднородном псевдоожиженном слое мелких частиц. Показано также, что в плотном слое и агрегатах псевдоожиженного $Nu_{\min} \approx 8{,}6$.

NOMENCLATURE

Ar,	Archimedes number, $\frac{gd^3}{\nu} \cdot \frac{\gamma_s - \gamma_g}{\gamma_g}$;
C_g,	specific heat of gas, kcal/kg degC;
C_0,	specific heat of gas at temperature t_0, kcal/kg degC;
C_ϑ,	specific heat of particles at temperature ϑ, kcal/kg degC;
C_{mix},	specific heat of mixture of particles and gas, kcal/kg degC;
d,	mean diameter of particles, m;
F_{sec},	cross-sectional area of a tube, m^2;
M_{sur},	weight of gas passing through a bed in the form of bubbles, kg/s;
M_{ag},	weight of gas passing through aggregates of particles in a bed, kg/h;
m,	porosity of a bed, m^3;
N,	fluidization number $\frac{W_{\text{sup}}}{W_{\text{of}}}$;
Nu,	Nusselt number;
Re,	Reynolds number $\frac{Wd}{\nu}$;
St,	Stanton number;
t_0,	initial gas temperature, degC;
t_1,	gas temperature after the first row of particles, degC;
$t_{1\text{mix}}$,	mean gas temperature after the first mixing, degC;
$t_{n\text{mix}}$,	mean gas temperature after the nth mixing, degC;
$(\Delta t)_{mn}$,	logarithmic mean temperature head, calculated by mean gas temperature, degC;
V_{sur},	volume of gas passing through a bed in the form of bubbles, m^3/s;
V_{ag},	volume of gas passing through aggregates of particles in a bed, m^3/s;
W_{of},	linear velocity of gas at onset of fluidization, m/s;
W_{sup},	superficial gas velocity, m/s.

Greek symbols

α,	total heat-transfer coefficient, kcal/m^{2}h degC;
α_{cond},	heat-transfer coefficient by molecular conduction, kcal/m^{2}h degC;
α_{conv},	heat-transfer coefficient by convection, kcal/m^{2}h degC;
α_r,	heat-transfer coefficient by radiation, kcal/m^{2}h degC;
α_{min},	minimum value of heat-transfer coefficient, kcal/m^{2}h degC;
γ_s,	specific weight of solid particles, kg/m^3;
γ_g,	specific weight of gas, kg/m^3;
δ,	thickness of gas shell round a solid particle, m;
ϑ,	temperature of particles, degC;
λ_g,	thermal conductivity of gas, kcal/m h degC;
λ_{conv},	convective component of effective thermal conductivity, kcal/m h degC;
λ_{eff},	effective thermal conductivity, kcal/m h degC;
ν,	coefficient of kinematic viscosity, m^2/s.

Subscripts

min,	minimum value;
of,	onset of fluidization;
eff,	effective value;
b,	fluidized bed.

THE problem of heat-transfer coefficients between solid particles and a gas in fluidized and packed beds may be called with good reasons both complex and confused. Suffice to say that all modern methods of experimental determination of this heat-transfer coefficient which produces values of the Nusselt number for fine particles $Nu \ll 2$ are unsound. As is given below, minimum true values of Nu for particles of any diameter in a bed are always more than two. For a packed bed and a fluidized one at the onset of fluidization $Nu_{\min} \approx 8{\cdot}6$.

Complete discrepancy between experimental (effective) and true values of Nu is caused mainly by the fact that for most particles in a bed the actual temperature drop is often a factor of from dozens to hundreds of times less than that determined experimentally by the shape of the temperature curve of the whole flow. Because of the very complicated but not quite random character of gas distribution in a fluidized bed, it is not yet possible to determine the true mean temperature drop in a bed according to experimental data. Measurements on models of a fluidized or packed bed and simplified analytical calculations, which will be considered below, may give approximate information on true heat-transfer coefficients.

In the general case the heat-transfer coefficient between solid particles and their surrounding fluid may be considered as a sum of three components: conductive (α_{cond}), convective (α_{conv}) and radiant (α_r)

$$\alpha = \alpha_{\text{cond}} + \alpha_{\text{conv}} + \alpha_r. \tag{1}$$

Thus, molecular and molar transfers are considered separately, i.e. α_{conv} does not account for transfer by molecular heat conduction of the fluid.

The conductive component α, caused by molecular heat conduction, is very large for fine particles. The heat-transfer coefficient of such particles is known (though often forgotten) to sharply increase with decrease in their diameter. This follows directly from the so-called Nusselt law which produces the limited minimum value of the heat-transfer coefficient for a spherical particle, assuming in this case heat to be transferred by molecular conduction alone through a gas sphere surrounding the particle. The outside diameter of this gas sphere is assumed to be infinitely large, i.e. the thickness of the layer which the predicted temperature drop is applied to, is infinite. This minimum value of the heat-transfer coefficient $\alpha = 2\lambda/d$ may be obtained by the equation of stationary heat conduction of a spherical wall.

In fact, for a spherical wall

$$Q = \frac{2\pi\lambda_g \Delta t}{(1/d_1) - (1/d_2)} = \alpha \Delta t \pi d_1^2$$

hence

$$d = \frac{2\lambda_g}{d_1(1 - d_1/d_2)} \tag{2}$$

where d_1 and d_2 are the particle diameter and the external diameter of fluid layer round the particle, respectively. In the case when $d_2 \to \infty$ we obtain the minimum value of heat-transfer coefficient from (2)

$$\alpha_{\min} = \frac{2\lambda_g}{d_1}. \tag{3}$$

Hence, the conductive component of the heat-transfer coefficient increases inversely as the particle diameter, i.e. infinitely large values of the heat-transfer coefficient may be obtained simply by decrease in the particle size. Thus, when the particle diameter decreases, intensification of the heat transfer is obtained not only by a larger heat-transfer surface per unit weight of a heated or cooled material but by increase in the heat-transfer coefficient itself.

Further increase in α_{cond} is obtained provided that the thickness of a fluid layer which the predicted temperature difference is applied to, decreases. This occurs in a fixed bed or in a fluidized one at the onset of fluidization. In this case the heat-transfer fluid layer between particles is very thin because of high volumetric concentration of solid particles. For instance, if the arrangement of particles is assumed to be cubic for the dense phase of a fluidized bed, the average thickness of a fluid "shell" round the particle will be

$$\delta = \frac{d^3 - \pi d^3/6}{\pi d} = 0{\cdot}152d.$$

In this case according to equation (2)

$$\alpha_{\text{cond}} = \frac{2\lambda_g}{d(1 - d/1{\cdot}304d)} = \frac{2\lambda_g}{0{\cdot}233d} \tag{4}$$

hence $Nu_{\min} \approx 8{\cdot}6$.

Passing to the convective component α, it should be kept in mind that this component is caused by molar transfer alone. Fluid velocity typical for the processes in heat exchangers with fixed (solid, dense) and fluidized bed is low: it ranges between several metres per second to a fraction of a metre per second. Consequently α_{conv} of particles would be expected to be small. However, this is not so. This becomes evident if the same problem of steady state heat conduction of a spherical wall is considered for effective heat conduction but not for molecular. The local coefficient of effective heat conduction

$$\lambda_{\text{eff}} = \lambda_g + \lambda_{\text{conv}} \tag{5}$$

also accounts for convective transfer λ_{conv}. If convective streams penetrated to the very surface of particles and λ_{conv} were constant for all the points, we should obtain

$$\alpha_{\text{cond}} + \alpha_{\text{conv}} = \frac{2\lambda_{\text{eff}}}{d(1 - d_1/d_2)}.$$

But in reality, in the presence of a laminar layer or sublayer, the convective component of the heat-transfer coefficient of a solid particle will be less than that from the above formula since near the particle surface λ_{conv} falls to zero, and the heat-transfer coefficient depends mainly on λ_{eff} of the fluid in the immediate vicinity of the particles. Even if molar transfer occurs outside this laminar "shell" is so intense that λ_{eff} there approaches infinity, the heat-transfer coefficient of a particle will not always differ greatly from $\alpha_{\min}$. The assumption that outside the laminar shell $\lambda_{\text{eff}} \to \infty$ is equivalent to the assertion, that the whole temperature head Δt is applied to this shell. Thus, this assumption leads to the previous problem of steady state heat conduction of a spherical wall δ in thickness, with heat-conduction coefficient λ_g. Then, for instance, when the laminar sublayer thickness $\delta = 2d$, the heat-transfer coefficient $\alpha = 1{\cdot}25\,\alpha_{\min}$, or it is really slightly greater than the minimum. This result may be expressed in another way: heat transfer of a particle is slightly influenced by fluid vortex diffusivity outside the spherical layer of two particle diameters in thickness. Relative thickness of a laminar layer (sublayer) round particles increases while passing to fine particles with small free fall velocity. Thus, α_{conv} is very small for them. But in the dense phase of a fluidized bed and in a fixed bed, where thickness of a layer between solid particles is small, disturbances of the stream should produce great increase in α owing to the convective component. Even now there is no reliable information on α_{conv} of particles inside the bed. However, recently Wardsworth's data appeared on heat-transfer coefficients of a sphere inside an infinite fixed bed of such spheres with ideal rombohedral arrangement and with the flow round spheres at an Re of some several tens of thousands, relative to the diameter of the sphere. The sphere diameter was about 100 mm. According to this data at $Re = 7000$–$50\,000$,

$$St \approx Re^{-0{\cdot}27}$$

or

$$\alpha_{\text{conv}} \approx 3600\, c_g \gamma_g \nu^{0{\cdot}27}\, W_{\text{sup}}^{0{\cdot}73}\, d^{-0{\cdot}27}. \tag{6}$$

If in the first approximation this relation is extended to the region with small Re and loose packing (the region with small number and lower efficiency of points of turbulizations), corresponding to the onset of fluidization, then apparently high values of α_{conv} will be obtained for the new conditions. For conditions $W_{\text{sup}} = W_{\text{of}}$, according to Leva's data

$$W_{\text{of}} \approx 9{\cdot}23 \times 10^{-3} K d^{1{\cdot}82} \nu^{-0{\cdot}88} \left(\frac{\gamma_{\text{p}} - \gamma_g}{\gamma_g}\right)^{0{\cdot}94} \tag{7}$$

where the correction factor K

$$K = 1{\cdot}775 Re^{-0{\cdot}272} \tag{8}$$

at $Re = 10$–200. From (6–8)

$$\alpha_{\text{conv}} = 341 c_g \gamma_g\, d^{0{\cdot}618} \nu^{-0{\cdot}019} \left(\frac{\gamma_p}{\gamma_g} - 1\right)^{0{\cdot}54}. \tag{9}$$

Table 1 gives very high numerical values of α_{conv} and Nu_{conv} which are calculated by (9) for the material, fluidized by air with $\gamma_p = 2500$ at $t = 100°C$ near the particle.

Table 1

d	10^{-4}	5×10^{-4}	10^{-3}	2×10^{-3}
α_{conv}	45·6	122·5	188	288·5
Nu_{conv}	0·165	2·22	6·82	20·9

The radiant component of heat transfer between a particle and a gas (but not between a particle and the containing wall or other particles) is usually negligibly small because of a great diathermic property of gases and small thickness of gas layers round the particle.

Thus, we have stated that even the minimum conductive heat-transfer coefficient of particles in a bed is several times larger than it was considered before, at $Nu_{\text{min}} = 2$. Thus, the divergence between great theoretical and often very small practical (effective) values of the Nusselt numbers seems to become more inexplicable.

Attempts at explaining the small experimental values of Nu for particles in a fluidized bed were made several times. Richardson and Ayers [10] suggested an original explanation which we consider erroneous. They think that a particle entering the effective heating zone of about 0·1 inches in height from the entry distributing manifold carries a gas shell into which heat begins to penetrate by conduction. After the particle leaves the heating zone, heat within the shell propagates both into the particle and in the opposite direction—into the main fluid body. The result is that only a small quantity of heat which penetrated into the shell, reaches the particle. However, the assumption is unjustified here that a particle can move in a fluidized bed without changing the gas in its "shell", especially when porosity is rather small ($m = 0·5$–$0·64$) as in Richardson and Ayer's experiments. Where then does the main gas stream really flow? It is interesting to note that low effective Nusselt numbers (many times less than two) are also observed in a fixed bed where motion of particles is absent completely.

Investigators have found numerous more or less serious sources of errors in the works of other scientists [1, 4, 5, 7, 9, 13] but even this did not throw light on the main reason for the extremely small effective Nusselt numbers.

Some of the early scientists who investigated fluidization (Toomey and Johnstone [11], Warmsley and Johnson [13]) have come nearest to the verisimilar explanation of this effect. They considered that the root of the trouble lay in non-uniformity of gas distribution through a bed. At the same time they simplified this effect, attributing it to the process in which a portion of gases passes through the whole bed without heat transfer between gas and particles. Toomey and Johnstone [11] supposed that the whole surplus gas over the quantity, necessary for minimum fluidization, passed through the bed. Already Warmsley and Johnson [13] have revealed the discrepancy between this conception and experimental data. Existence of rather intensive transfer of gas between bubbles (the discontinuous phase) and aggregates of particles (the continuous phase) has been proved by further investigations into the aerodynamics of fluidized beds. It seems to completely undermine the conclusions based on the previous conception of the bypassing of a portion of gas. However, we believe that the considerations below serve to convince that the presence of particle aggregations and incomplete local bypassings of the gas ("micro-breaks") should cause the low effective Nusselt numbers even under the conditions of intensive gas transfer between the "phases".

It is well known that, in the presence of fluidization by gases, a bed is non-uniform and aggregation of particles takes place. In the interior of aggregates the filtration velocity is close to that of minimum fluidization W_{0f}. This tendency is partially explained theoretically and confirmed experimentally by the works of Baumgarten and Pigford [6], Yasui [14] and Dotson [8]. Gas bubbles bypass the aggregates. As is known, the aggregates in a fluidized bed (the continuous phase) are unstable. At one moment they appear, only to disappear, new aggregates being made up from the broken down aggregates, these in turn abstracting gas from the bubbles (discontinuous phase). This is one of the reasons why gas transfer takes place between the discontinuous phase and the continuous one. Furthermore, the walls of bubbles are permeable to a gas.

The complex resulting picture of heat transfer between a material and a gas with gas exchange between aggregates and bubbles, may be reduced to an equivalent model conception in which a gas flow through an aggregative fluidized bed will be considered as a combination of "microbreaks" of the surplus gas to the height of one or several horizontal rows of particles followed by complete lateral mixing of the whole gas beyond these rows, new microbreaks and so on and so forth. The limited heterogeneity of the layer, corresponds in this system to ideal mixing after each layer (Fig. 1a), when a huge degree of heterogeneity is associated with many rows (Fig. 1b). It is assumed that in the aggregate itself the filtration velocity is close to that of minimum fluidization.

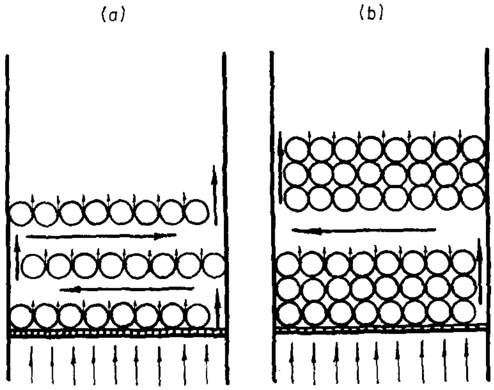

FIG. 1. "Microbreak" of gases inside the aggregative fluidized bed.

It is worth remarking on the rather deep cooling of the gas passing through the aggregate in the first (lower) row of solid particles. Consider the heat transfer of one particle in the first row. The volume of the gas flowing round the particle per hour is equal to $3600 \times W_{of} \times d^2$ with cubic packing of particles in the aggregate. Equating the quantity of heat released by the gas and that of heat taken by the particle, we have

$$\pi d^2 \alpha_{of} (\Delta t)_{mn} = 3600 W_{of} d^2 C_g \gamma_g (t_0 - t_1) \qquad (10)$$

or

$$(\Delta t)_{mn} = A(t_0 - t_1)$$

where

$$A \equiv \frac{3{\cdot}6 \times 10^3 C_g \gamma_g W_{of}}{\pi \alpha_{of}}. \qquad (10a)$$

According to (7) and (8)

$$W_{of} = 3{\cdot}95 \times 10^{-2} d^{1{\cdot}217} \nu^{-0{\cdot}478} \left(\frac{\gamma_s}{\gamma_g} - 1\right)^{0{\cdot}739} \qquad (11)$$

Calculating the value of A for the case of fluidization of material by air at $\gamma_s = 2500$ kg/m^3. Physical parameters for air are taken at 100°C.

Accounting for the conductive component α_{of} alone, we assume

$$\alpha_{of} \approx \frac{8{\cdot}6 \lambda_g}{d}.$$

Substituting numerical values of physical constants in equation (11) and in the expression for A, we obtain

$$W_{of} = 2{\cdot}175 \times 10^3 d^{1{\cdot}217} \qquad (11a)$$

and

$$A = 2{\cdot}4 \times 10^6 \times d^{2{\cdot}217}.$$

Numerical values of A, calculated for particles with different diameters are given in Table 2. This table indicates the proportion of the original temperature head $(t_0 - \vartheta)$ which goes to the temperature head after the first row $(t_1 - \vartheta)$ provided that temperature of the particles ϑ in the first row is practically constant (ϑ = const.). This occurs in drying during the period of constant velocity and always when mixing of material in a bed is fine enough.

Table 2

d_{1s}	A	$(t_1 - \vartheta)/(t_0 - \vartheta)$
10^{-4}	$3{\cdot}31 \times 10^{-3}$	0
3×10^{-4}	$3{\cdot}8 \times 10^{-2}$	$3{\cdot}54 \times 10^{-12}$
5×10^{-4}	0·117	2×10^{-4}
10^{-3}	0·538	0·155

In this case

$$(\Delta t)_{mn} = \frac{t_0 - t_1}{2{\cdot}3\lg(t_0 - \vartheta/t_1 - \vartheta)}$$

hence, accounting for equality (10a).

$$\frac{t_1 - \vartheta}{t_0 - \vartheta} = (10^{1/2\cdot 3.A})^{-1}$$

From Table 2 it follows that the portion of the temperature head remaining in the aggregate after the first row of particles is often extremely small. For particles of the order of 500 μ and finer practically complete cooling (heating) prevails of aggregate gases to the temperature of the material. Allowing for α_{conv} and unsteady state heat-transfer conditions, this conclusion should be applied to coarser particles as well.

What is the real rate of change in mean temperature of a gas with height in an aggregation fluidized bed? Apply the concept of the two-phase theory of fluidization to microbreak conditions. Surplus gas, passing in the form of discontinuous phase, is approximately

$$V_{\mathrm{sur}} = W_{\mathrm{sup}}F_{\mathrm{sec}} - W_{\mathrm{of}}F_{\mathrm{sec}}. \qquad (12)$$

Equation (12) agrees well with the other conception of the same theory—namely that on filtration velocity within aggregates equal to W_{of}—only in the case where the portion of the cross section, occupied by the stream of by-passing gas, is negligibly small, i.e. the velocity of the discontinuous phase is sufficiently large compared with W_{of}. This assumption is valid for approximate calculations. Then, the portion of gas passing through the aggregate is given by

$$V_{\mathrm{ag}} = V - V_{\mathrm{sur}} = W_{\mathrm{sup}}F_{\mathrm{sec}} - V_{\mathrm{sur}} = W_{\mathrm{of}}F_{\mathrm{sec}}. \qquad (13)$$

With regard to the sharp change in the gas temperature when particles are passing through the aggregate in a heat exchanger, the question arises as to what temperature gases this condition refers. Since in the major part of the aggregate, the gas temperature is close to ϑ, we have every reason to calculate W_{of} and W_{sup} by ϑ. Thus, the weights of the surplus gas passing through the aggregates will be equal to

$$M_{\mathrm{sur}} = (W_{\mathrm{sup}} - W_{\mathrm{of}})\gamma_{\vartheta} \cdot F_{\mathrm{sec}}$$

and

$$M_{\mathrm{ag}} = W_{\mathrm{of}} \cdot \gamma_{\vartheta} \cdot F_{\mathrm{sec}}$$

respectively (Fig. 1a).

According to the presented scheme of "microbreaks", after the first row of particles, M_{ag} of the gas cooled to ϑ is mixed with M_{sur} at initial temperature t_0. The case considered is the most favourable one for the rapid decrease in the mean temperature of a gas in an aggregative bed (minimum height of aggregates, complete lateral mixing of gas after every row of particles).

The temperature of a mixing is defined by the balance of mixing

$$M_{\mathrm{sur}}t_0C_0 + M_{\mathrm{a}}\vartheta\theta C_{\vartheta} = (M_{\mathrm{sur}} + M_{\mathrm{ag}})t_{1\mathrm{mix}}C_{\mathrm{mix}}.$$

Neglecting the change in specific heat, we have

$$M_{\mathrm{sur}}t_0 + M_{\mathrm{ag}}\vartheta = (M_{\mathrm{sur}} + M_{\mathrm{ag}})t_{1\mathrm{mix}}.$$

Hence, the mean temperature of the gas after the first mixing is easily obtained

$$t_{1\mathrm{mix}} = \frac{(N - 1)t_0 + \vartheta}{N}$$

where $N = W_{\mathrm{sup}}/W_{\mathrm{of}}$ is the fluidization number.

Similarly for the nth mixing after the nth microbreak

$$t_{n\mathrm{mix}} = \frac{(N - 1)t_{n-1\mathrm{mix}} + \vartheta}{N}. \qquad (14)$$

Expanding (14) we obtain

$$t_{n\mathrm{mix}} = \frac{(N-1)^n t_0 + (N-1)^{n-1}\vartheta + (N-1)^{n-2}N\vartheta + \ldots (N-1)^{n-K}N^{K-1}\vartheta + \ldots N^{n-1}\vartheta}{N^n}.$$

According to (14) for extreme cases, when N, t_0 and ϑ are given, the change in the mean gas temperature (t_{mix}) may be calculated by the height of the aggregative fluidized bed. In Table 3 the results of a similar calculation are given for the case when $t_0 = 400$°C, $\vartheta = 100$°C, and $N = 2$ and 20 (certainly, a fluidized bed is possible at $N = 20$ only provided that the Archimedes numbers are sufficiently small).

Table 3 indicates a fairly slow drop in the mean gas temperature across the cross section accompanying a microbreak, in spite of intensive gas exchange between the discontinuous phase

Table 3

N	2	30
θ	100	100
t_0	400	400
$t_{1\mathrm{mix}}$	50	385
$t_{2\mathrm{mix}}$	175	371
$t_{3\mathrm{mix}}$	137·5	358
$t_{4\mathrm{mix}}$	118·8	345
$t_{5\mathrm{mix}}$	109·4	333
$t_{6\mathrm{mix}}$	104·7	321
$t_{7\mathrm{mix}}$	102·3	310
$t_{8\mathrm{mix}}$	101·2	299
$t_{9\mathrm{mix}}$	100·6	289
$t_{10\mathrm{mix}}$	100·3	280
$t_{11\mathrm{mix}}$	100·15	271
$t_{12\mathrm{mix}}$	100·07	263

phase and the continuous one (complete radial mixing of a gas after every row of particles), especially when the fluidization number is large. It will be shown by numerical example that extremely low Nu_{eff} should be expected even when $N = 2$ and large true heat-transfer coefficients ($Nu = 8{\cdot}6$) are large. N_{eff} is usually calculated by the height of the bed where the mean gas temperature reaches equilibrium value within to 0·1 degC. According to Table 3 this height consists of twelve rows of particles. Assume $d = 10^{-4}m$. The part of the cross section, where the discontinuous phase occurs, is neglected. Then, the surface of particles in a horizontal row with their cubic packing is [2]

$$F_1 = \frac{6(1 - m)V_6}{d} = \frac{6(1 - 0{\cdot}476)\, dF_{\mathrm{sec}}}{d}$$
$$= 6 \times 0{\cdot}524 F_{\mathrm{sec}}.$$

The results of heat transfer in twelve rows are described by

$$3600 W_s F_{\mathrm{sec}} \gamma_{100} C_{100}(t_0 - t_{12\mathrm{mix}}) = \alpha_{\mathrm{eff}}(\Delta t)_{mn}\, 12F_1, \quad (15)$$

where α_{eff} is the effective heat-transfer coefficient, $(\Delta t)_{mn}$ is the logarithmic temperature of a gas.

In this case

$$(\Delta t)_{mn} = \frac{(t_0 - \vartheta) - (t_{12\mathrm{mix}} - \vartheta)}{2{\cdot}3 \lg (t_0 - \vartheta / t_{12\mathrm{mix}} - \vartheta)}$$
$$= \frac{300 - 0{\cdot}07}{2{\cdot}3 \lg (300/0{\cdot}07)} = 36 \text{ per cent.}$$

According to (15) the effective Nusselt number

$$Nu_{\mathrm{eff}} = \frac{\alpha_{\mathrm{eff}}\, d}{\lambda_g}$$
$$= \frac{3600 W_s \gamma_{100} C_{100}(t_0 - t_{12\mathrm{mix}})\, d}{12 \times 6 \times (t\Delta)_{mn} \times 0{\cdot}524 \lambda_{100}}. \quad (16)$$

Here the filtration velocity is $W_s = NW_{\mathrm{of}} = 2W_{\mathrm{of}}$, and the velocity of minimum fluidization by (11a) is

$$W_{\mathrm{of}} = 2{\cdot}175 \times 10^3 (10^{-4})^{1{\cdot}217}$$
$$= 2{\cdot}95 \times 10^{-2} \text{ m/s}.$$

Substituting numerical values of the quantities entering (16) we obtain

$$Nu_{\mathrm{eff}} = \frac{3600 \times 2 \times 2{\cdot}95 \times 10^{-2} \times 0{\cdot}946 \times 0{\cdot}241(400 - 100{\cdot}07) \times 10^{-4}}{12 \times 6 \times 36 \times 0{\cdot}524 \times 2{\cdot}76 \times 10^{-2}}$$
$$= 3{\cdot}88 \times 10^{-2}.$$

Thus, in our case Nu_{eff} is $8{\cdot}6/3{\cdot}88 \times 10^{-2} \approx 220$ times less than true Nu.

Evidently, for larger fluidization numbers the smaller value of Nu_{eff} would be obtained, as it may be judged by the slow approaching of the mean gas temperature to the equilibrium one at $N = 20$ (Table 3).

In real aggregative fluidized beds of fine particles, heat transfer may be worse since the height of aggregates is equal to more than one row of particles, and complete radial mixing of a gas does not occur after every row.

The above approximate analysis of the effect of gas microbreaks on heat transfer in a fluidized bed leads to the conclusion that sharp changes in the effective heat-transfer coefficient of particles should be expected not only when strongly pronounced cohesion of particles (e.g. under the influence of molecular forces) takes place, but when changes in aggregation are visually unnoticed (e.g. under the influence of weak static electrification). In these cases unstable aggregates may change in size and in existence time and, consequently, the intensity of gas exchange between the discontinuous and continuous phases or the number of complete radial mixings can change as well (according to the schematic theoretical model). This considerably (if not mainly) explains the discrepancy between the data obtained by various investigators on effective heat-transfer coefficients.

It is not out of place to note the value of accumulating experimental data on effective heat-transfer coefficients α_{eff} of particles in a fluidized bed. Owing to the arbitrariness and inconsistencies between $\alpha_{\mathrm{eff}}(Nu_{\mathrm{eff}})$ and true $\alpha(Nu)$, it is important to know the effective values. The relation of Nu/Nu_{eff} characterizes the degree of imperfection of fluid distribution in a heat exchanger and theoretical prospects of heat-transfer improvement. As to the direct application of α_{eff} to the design of heat exchangers with fluidized beds, somewhat exact calculations are possible only under the conditions similar to those when $\alpha_{\mathrm{eff}}(Nu_{\mathrm{eff}})$ is obtained, including the conditions of initial fluid distribution, electrification, etc. In the opposite case when the value of *Re* alone is known, predicted and real operating values of α_{eff} may differ by a factor of two or three. This possibility should be born in mind and large reserves be left when designing.

Evidently, various means to increase uniformity of fluidization may be used for improvement of heat transfer between a gas and particles in a fluidized bed. These methods are as follows: hindering of a bed by permeable baffles, the effect of strong sound or ultrasonic waves; vibration of gas distributors or elements; immersed in a bed and improvement of uniformity of the initial distribution of a gas.

Baffles break and hinder aggregates and permit the increase of velocity of the gas in aggregates.

Various oscillations and vibrations are considered to affect heat transfer between a gas and a bed since particles then move faster and break up the aggregates. Intensification of heat transfer between particles and a gas, caused by increase in relative velocity of flowing and taking place during the unsteady period of formation of a boundary layer (sublayer), is probably only of secondary importance in this case. The boundary layer would be undeveloped even under steady conditions of flow and of heat transfer for in the dense phase of fluidization the particles are closely packed.

The problem of arranging uniform initial gas distribution is essential. The point is that the very problem of increasing the effective heat-transfer coefficient of particles in a fluidized bed becomes really urgent when units with a shallow bed are designed units which are desirable because of the small pressure drop. On the other hand the whole shallow bed is within the influence of the inlet gas distributor. "Classical" non-uniformity of fluidization with large bubbles and dense aggregates cannot develop completely in a shallow bed. However, here the danger of channelling (a form of microbreak) is great because of inadequate design of the gas distributor. And in case of bad mixing of particles near a gas distributor a zone of overheated material arises, the zone of gas cooling is extended and $\alpha_{\mathrm{eff}}(Nu_{\mathrm{eff}})$ further decreases.

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Abstract—A model conception is introduced of "micro-intersections" of gas in fluidized beds. Using this theoretical model it is shown how extremely high true heat-transfer coefficients may be found in association with very small effective ones when non-uniform fluidization occurs.

It is also shown that $Nu_{\min} \approx 8{\cdot}6$ in a solid bed and within aggregates in a fluidized bed.

Résumé—L'auteur présente un modèle pour les "microintersections" du gaz dans des lits fluidisés. En utilisant ce modèle théorique, il montre que l'on peut trouver des coefficients de transport de chaleur vrais très élevés associés à des coefficients effectifs de transport de chaleur très petits quand une fluidisation non-uniforme se produit.

Il montre également que $Nu_{\min} \approx 8{,}6$ dans un lit fixe et dans les aggrégats d'un lit fluidise.

Zusammenfassung—Es wird die Modellvorstellung von einem "Mikrodurchbruch" von Gas in Fliessbetten eingeführt. Mit Hilfe dieses theoretischen Modells wird gezeigt, wie extrem hohe errechnete Wärmeübergangskoeffizienten sehr kleinene ffektiven Koeffizienten bei nicht gleichmässiger Fluidisierung gegenüberstehen. Im Festbett und innerhalb von Zusammenballungen im Fliessbett wird $Nu_{\min} \approx 8{,}6$.